

# Jornadas de Análisis Geométrico en Valparaíso

Títulos y Resúmenes

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# 1 Uniqueness of Semigraphical Translators of Mean Curvature Flow.

**Mariel Saez**

**Jueves 10:00hs.**

**Institución:** Facultad de Matemáticas. Pontificia Universidad Católica de Chile.

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**Resumen:** In this talk we prove the uniqueness of pitchfork and helicoid translators of the mean curvature flow in  $\mathbb{R}^3$ . This solves a conjecture by Hoffman, White and Martin. The proof is based on an arc-counting argument motivated by Morse-Radó theory for translators and a rotational maximum principle.

This is joint work with F. Martin and R. Tsiamis.

# 2 A Morse-Theoretic Glance at Phase Transitions Approximations of Mean Curvature Flows.

**Pedro Gaspar**

**Jueves 11:00hs.**

**Institución:** Facultad de Matemáticas. Pontificia Universidad Católica de Chile.

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**Resumen:** In this talk, we describe some recent results concerning eternal gradient flows for a phase-transitions approximation of the area functional in a compact Riemannian manifold. These gradient flows are solutions to Allen-Cahn type equations and provide a regularization for the mean curvature flow of hypersurfaces in such ambient spaces. Using Morse-theoretical considerations, we construct eternal solutions of the Allen-Cahn equation that connect distinct equilibria in generic compact manifolds. We describe a more complete picture for the space of such connections in the concrete example of a round 3-sphere under a low-energy assumption, and indicate how these solutions can be used to produce geometrically interesting eternal MCFs.

This is joint work with Jingwen Chen (University of Pennsylvania).

### 3 Bifurcacion Global y Métricas de Q-Curvatura Constante.

Jimmy Petean

Jueves 14:00hs.

**Institución:** Centro de Investigación en Matemática,  
Guanajuato, México.

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**Resumen:** Consideraremos el problema de multiplicidad de métricas de Q-curvatura constante en el producto Riemanniano de variedades de Einstein. Asumiendo que uno de los factores admite una función isoparamétrica el problema se reduce a encontrar soluciones positivas a una ecuación diferencial ordinaria de orden 4. Obtendremos multiplicidad de soluciones a esta ecuación usando técnicas de bifurcación global. De esta forma probaremos que el número de métricas de Q-curvatura constante crece al variar el radio de los factores del producto Riemanniano.

### 4 TBA.

María del Mar González

Viernes 10:00hs.

**Institución:** Departamento de Matemáticas Universidad Autónoma de Madrid,  
España.

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**Resumen:**

## 5 Delaunay-type compact equilibria in a nonlocal isoperimetric problem.

Andrés Zúñiga

Viernes 11:00hs.

**Institución:** Instituto de Ciencias de la Ingeniería Universidad de O'Higgins, Rancagua, Chile.

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**Resumen:** We deal with the *liquid drop model*, introduced by G. Gamow (1930) and N. Bohr- J.A. Wheeler (1939) in nuclear physics to describe the structure of atomic nuclei [1], [2]. This problem consists of finding a surface  $\Sigma = \partial\Omega$  in  $\mathbb{R}^3$  that is a critical point for the following energy:

$$\mathcal{E}(\Omega) = \text{Per}(\Omega) + \frac{1}{2} \iint_{\Omega \times \Omega} \frac{1}{|x - y|} dx dy,$$

for regions  $\Omega \subset \mathbb{R}^3$  under a volume constraint  $|\Omega| = m$ . The associated Euler-Lagrange equation is

$$H_{\Sigma}(x) + \int_{\Omega} \frac{dy}{|x - y|} = \lambda, \quad \forall x \in \Sigma, \quad \text{and} \quad |\Omega| = m,$$

where  $\lambda$  is a constant Lagrange multiplier. Round spheres enclosing balls of volume  $m$  are always solutions. They are minimizers for sufficiently small  $m$ . Since the two terms in the energy compete, finding non-minimizing solutions can be challenging. We find a new class of solutions with large volumes, consisting of *pearl collars* with an axis located on a large circle, with a shape close to a *Delaunay's unduloid surface* with constant mean curvature.

This is joint work with Manuel del Pino (Department of Mathematical Sciences, University of Bath, Bath, UK) and Monica Musso (Department of Mathematical Sciences, University of Bath, Bath, UK).

## Referencias

- [1] Gamow, G. *Mass defect curve and nuclear constitution*. PROC. ROYAL SOC. A **126** (803): 1930.
- [2] Bohr, N., Wheeler, J.A. *The mechanism of nuclear fission*. PHYS. REV. **56**(426): (1939).

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## 6 Moduli space for complete, constant Q-curvature metrics.

Rayssa Cajú

**Viernes 14:00hs.**

**Institución:** Departamento de Ingeniería Matemática Universidad de Chile

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**Resumen:** The Q-curvature equation, a fourth-order elliptic partial differential equation with a critical exponent, is a prominent class of conformal equations, largely due to its connection with a natural concept of curvature.

In this talk, we study constant Q-curvature metrics conformal to the round metric on the sphere with finitely many point singularities. We show that the moduli space of solutions with finitely many punctures in fixed positions, equipped with the Gromov-Hausdorff topology, has the local structure of a real algebraic variety with formal dimension equal to the number of the punctures.

This is joint work with Jesse Ratzkin and Almir Silva Santos.