# Viscosity solutions of first order Hamilton-Jacobi equations in proper CAT(0) spaces

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We propose a novel notion of viscosity solutions to study first order Hamilton-Jacobi equations in a certain class of metric spaces called proper CAT(0) spaces.

A metric space $(X, d)$ is said to be a CAT(0) *space* if, roughly speaking, it is a geodesic space and of non-positive curvature in the sense of the triangle comparison theorem. They can be seen as a generalization of Hilbert spaces or Hadamard manifolds. Typical examples of CAT(0) spaces include Hilbert spaces, simply connected Riemannian manifolds that have everywhere non-positive sectional curvature, metric trees and networks obtained by gluing a finite number of half-spaces along their common boundary.

Although CAT(0) spaces are not manifolds in general, they carry a solid first order differential calculus resembling that of a Hilbert space. For example, a notion of tangent cone is well defined at each point of $X$. The tangent cone is the metric counterpart of the tangent space in Riemannian geometry or the Bouligand tangent cone in convex analysis. Furthermore, a notion of differential is well defined for any real-valued function $u : X \to \mathbb{R}$ that is Lipschitz and can be represented as a difference of two semiconvex functions (Lipschitz and DC functions in short). We propose to exploit all this additional structure that CAT(0) spaces enjoy to study stationary and time dependent first order Hamilton-Jacobi equation in them. In particular, we want to recover the main features of viscosity theory: *the comparison principle and Perron's method*.

In this talk, we give the main hypotheses we require for the Hamiltonian in this setting. Furthermore, we define the notion of viscosity solutions, using test functions that are Lipschitz and DC. Moreover, we show that we obtain the comparison principle using the variable doubling technique. Finally, we derive existence of the solution from the comparison principle using Perron's method in a similar manner as in the classical case of $X = \mathbb{R}^N$.