A continuous–time model of self–protection

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We propose an optimal linear insurance demand model, where the protection buyer can also exert a time-dynamic costly prevention effort to reduce her risk exposure. This is expressed as a stochastic control problem that consists in maximising an exponential utility of a terminal wealth. We assume that the effort reduces the intensity of the jump arrival process and interpret this as dynamic self-protection. We solve the problem by using a dynamic programming approach, and we provide a representation of the certainty equivalent of the buyer as the solution to a backward stochastic differential equation (BSDE). Using this representation, we prove that an exponential utility maximiser has an incentive to modify her effort dynamically only in the presence of a terminal reimbursement in the contract. Otherwise, the dynamic effort is actually constant, for a class of compound Poisson loss processes. If there is no terminal reimbursement, we solve the problem explicitly and identify the dynamic certainty equivalent of the protection buyer. This shows in particular that the Lévy property of the loss process is preserved under exponential utility maximisation. We also characterise the constant effort as the unique minimiser of an explicit Hamiltonian, from which we can determine the optimal effort in particular cases. Finally, after studying the dependence of the BSDE associated to the insurance buyer on the linear insurance contract parameter, we prove the existence of an optimal linear cover that is not necessarily zero or full insurance.