

# Optimal Control with Sweeping Dynamics and Mixed Constraints: Approaches for Necessary Conditions.

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In the pursuit of modeling elastoplastic mechanical systems, J.J. Moreau introduced the concept of a ‘sweeping process’ in the 1970s. These systems are distinguished by their dynamics, described through a discontinuous differential inclusion that can be articulated in terms of a cone, presenting a unique challenge. This presentation delves into the complexities of establishing necessary optimality conditions for optimal control problems involving these dynamic systems and an additional element: a mixed (control/state) constraint.

We will explore two distinct approaches to address these challenges. The first approach is grounded in a reformulation proposed in [dPFS19] and builds upon results presented in [BdP21] concerning necessary conditions for Optimal Control Problems subject to nonregular mixed constraints. However, it comes with anticipated drawbacks, notably the emergence of elements from  $(L^\infty)^*$  as multipliers and the absence of the maximization condition found in the classical Pontryagin Maximum Principle form of necessary conditions.

The second approach aims to address these limitations by adopting the methodology outlined in [dPFS19], where the sweeping term is approximated using Lipschitzian dynamics. While this approach improves upon certain aspects of the conclusions drawn from the first method, it introduces additional assumptions on both the mixed constraint and the dynamics. It is essential to note that this work is part of an ongoing research project, and for further details, we direct the reader to [CKSD22].

- [BdP21] J. A. Becerril and M. D. R. de Pinho. Optimal control with nonregular mixed constraints: An optimization approach. *SIAM Journal on Control and Optimization*, 59(3):2093–2120, 2021.
- [CKSD23] Karla L. Cortez, Nathalie T. Khalil, and Julio E. Solís-Daun. A sweeping process control problem subject to mixed constraints. *(In Press) IFAC-PapersOnLine*, 2023.
- [dPFS19] M d R de Pinho, MMA Ferreira, and GV Smirnov. Optimal control involving sweeping processes. *Set-Valued and Variational Analysis*, 27(2):523–548, 2019.