Interchange formula on non-separable Banach spaces with applications

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Let $(T,\mathscr{A},\mu)$ be a measure space, X a Banach space, a function $f\colon T\times X\to\overline{\mathbb{R}}$ and a space of measurable functions from T to X, called $\mathscr{X}$. The integral functional associated to f is

$$\begin{aligned}\mathscr{I}_f\colon \mathscr{X} &\to \overline{\mathbb{R}}\\ x &\mapsto \mathscr{I}_f(x) := \int_T f(t,x(t))d\mu.\end{aligned}$$

This class of functionals constitutes an important object for many areas of applied mathematics, especially for convex analysis, where R.T. Rockafellar has extensively studied it for normal integrands in finite-dimensional spaces, and later on separable Banach spaces. In this work, we study integral functionals defined on spaces of functions with values on general (non-separable) Banach spaces, providing an interchange formula between integration and infimum, which enables us to get explicit formulas for the conjugate and Clarke subdifferential of integral functionals and to give applications to calculus of variations. We refer the reader to [GPAV23] for more details.

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