Optimality Conditions for Control Problems Through Duality Theory

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In this presentation, we introduce both sufficient and necessary optimality conditions derived in [BH22] for a linear-convex optimal control problem with mixed constraints. To clarify, the term "linear-convex" denotes a structure where equality constraints are linear, while inequality constraints and the cost function are jointly convex in the state-control variables. A noteworthy aspect of these conditions is that differentiability in the functions defining the problem or regularity assumptions on the mixed constraints are not required. Specifically, the absence of differentiability mandates the expression of optimality conditions through convex subdifferentials and normal cones.

The broad applicability of these conditions arises from our proof methodology, wherein we formulate the control problem as a calculus of variations problem and leverage Rockafellar's duality theory for problems posed over arcs of bounded variation, as outlined in [Roc76]. Anticipating that the solution to this dual problem belongs to the class of functions of bounded variation, we infer a similar property for the costate of the original control problem. The proof of the necessary conditions also necessitates properties linking the convex subdifferential of the Lagrangian to the value function of the problem, as presented in [HW19]. Additionally, we delve into the regularity of the costate under the "bounded slope" constraint qualification.

In conclusion, we explore an ongoing effort to extend these results by deriving optimality conditions for fully convex impulsive optimal control problems subject to differential inclusions.

[BH22] Jorge Becerril and Cristopher Hermosilla. Optimality conditions for linear-convex control problems with mixed constraints. *J Optim Theory Appl*, 2022.

[HW19] Cristopher Hermosilla and Peter Wolneski. A characteristic method for fully convex bolza problems over arcs of bounded variation. *J Control Optim*, 2019.

[Roc76] Ralph Rockafellar. Dual problems of lagrange for arcs of bounded variation. *Calculus of Variations and Control Theory*, 1976.