Metric vs total dissipativity for measure differential equations

Giulia Cavagnari, Politecnico di Milano

Giuseppe Savaré, Bocconi University

Giacomo E. Sodini, Universität Wien

Contact: giulia.cavagnari@polimi.it

Measure differential equations are first order evolution equations in the Wasserstein metric space of probability measures $(\mathscr{P}_2(\mathbb{R}^d), W_2)$, driven by *probability vector fields* (PVFs). Taking inspiration from the Hilbertian theory, we can recover well-posedness by introducing a metric notion of dissipativity for PVFs which, together with growth-conditions, ensures the convergence of an explicit Euler scheme [CSS23b].

In this talk, we discuss the feasibility of an implicit Euler scheme for measure differential equations. Contrary to the Hilbertian case, it turns out that metric dissipativity does not imply contraction of the resolvent in the Wasserstein framework. By introducing the notion of total dissipativity, we are able to circumvent this difficulty by lifting the problem to the Hilbert space $L^2(\Omega;\mathbb{R}^d)$ of parametrizations [CSS23a].

Wasserstein gradient flows, i.e. when the probability vector field is given by the subdifferential of a displacement-convex functional, constitute particular examples of application.

[CSS23a] Giulia Cavagnari, Giuseppe Savaré, and Giacomo E. Sodini. A Lagrangian approach to totally dissipative evolutions in Wasserstein spaces. *arXiv:2305.05211*, 2023.

[CSS23b] Giulia Cavagnari, Giuseppe Savaré, and Giacomo Enrico Sodini. Dissipative probability vector fields and generation of evolution semigroups in Wasserstein spaces. *Probab. Theory Related Fields*, 185(3-4):1087–1182, 2023.

[CSS23c] Giulia Cavagnari, Giuseppe Savaré, and Giacomo Enrico Sodini. Extension of monotone operators and Lipschitz maps invariant for a group of isometries. *arXiv:2305.04678*, 2023.