Befriending $\mathscr{P}_2(\mathbb{R}^d)$: Viscosity solutions of centralized control problems in measure spaces

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We present a notion of viscosity solutions adapted to control problems in the space of probability measures over $\mathbb{R}^d$, endowed with the Wasserstein distance. The key idea is to avoid the definition of a gradient of applications going from $\mathscr{P}_2(\mathbb{R}^d)$ to $\mathbb{R}$, by working directly with directional derivatives. Modulo adjustments to cope with the absence of local compactness, our definition is then very similar to the finite-dimensional case. We show that such a viscosity notion supports a strong comparison principle, and that the value function of the control problem is the unique viscosity solution of the Hamilton-Jacobi-Bellman equation associated to the control problem. This work follows and generalizes [JJZ22], in which the underlying space is a compact manifold and the dynamic is independant of the measure. Additionally, we are able to consider weaker regularity on the terminal cost and the dynamic, and we take care to only use arguments relying on the geometry of the general tangent cone.

[JJZ22] Frédéric Jean, Othmane Jerhaoui, and Hasnaa Zidani. Deterministic optimal control on Riemannian manifolds under probability knowledge of the initial condition (to appear). 2022.

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