Optimal rotation in forest plantations: maximizing CO2 capture and the net benefit of the wood volume.

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Forest plantations are economically and environmentally relevant, as they play a key role in timber production and carbon capture. It is expected that the future climate change scenario affect forest growth and modify the rotation age for timber production. However, mathematical models on the effect of climate change on the rotation age for timber production remain still limited. Therefore, determining optimal management strategies is a challenge in forestry management. In the first place, A model was formulated from a system of Ordinary Differential Equations (ODEs) governed by the state variables live biomass volume, intrinsic growth rate, and area affected by fire. Then, four control variables were associated with the system, representing forest management activities, which are felling, thinning, reforestation, and fire prevention. and two optimal control problems are formulated from two different objective functionals. One of them is ecological in nature, focused on maximizing carbon capture, and the other is bioeconomic focused on maximizing the net present value associated with the profit from the volume of wood obtained from felling. The existence of optimal control solutions was demonstrated and the solutions of the optimal control problems were also characterized using Pontryagin's Maximum Principle. In the case of the ecological problem, the model solutions are approximated numerically by the fourth-order Runge-Kutta method. To verify the efficiency of the model, parameters for three scenarios were considered: a realistic one that represents current forestry activities based on previous studies for the exotic species Pinus radiata D. Don, another pessimistic, which considers significant losses in forest productivity; and a more optimistic scenario which assumes the creation of new forest areas that contribute with carbon capture to prevent the increase in global temperature. For the bioeconomic problem, the solutions of the model were approximated numerically by the Forward-Backward Sweep method. To validate the model, two scenarios were considered: a realistic scenario that represents current forestry activities for the exotic species Pinus radiata D. Don, and a pessimistic scenario, which considers environmental conditions conducive to a higher occurrence of forest fires. Finally, the rotation ages in both problems are compared for the species under study. Simulations are given to reinforce the theoretical results.

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