A numerical algorithm for mirror sweeping processes

Aldo Gutiérrez, Universidad de Chile

Emilio Vilches, Universidad de O'higgins

Contact: agutierrez@dim.uchile.cl

In this talk, we explore a particular case of the Degenerate Sweeping Process, an evolution problem described by a differential inclusion, in an infinite-dimensional Hilbert space. Our instance of the dynamic is the following:

$$\begin{cases} \dot{x}(t) \in -N(C(t); \mathscr{A}(x(t))) + f(t, x(t)) & \text{a.e. } t \in I, \\ x(T_0) = x_0. \end{cases} \tag{MSP}$$

The operator $\mathscr{A}$ is represented by the gradient of a lower semicontinuous, convex and proper function φ, inspired by the mapping used in the mirror descent algorithm. We refer to this dynamic as the *Mirror Sweeping Process*.

We construct a family of approximated solutions of (MSP), in the case where the moving sets are Lipschitz with respect to the excess, through a catching-up-like algorithm governed by the *approximator* operator (P_C^{φ}) introduced in [KMM98]. The step of the scheme is described as

$$x_{i+1}^n = P^{\varphi}_{C(t_{i+1}^n)}\left(x_i^n + \int_{t_i^n}^{t_{i+1}^n} f(s, x_i^n)\,ds\right).$$

We show that this family converges uniformly to the one and only solution of the Mirror Sweeping Process, which does not require neither compactness of the sets nor linearity of the operator.

[KMM98] M. Kunze and M. D. P. Monteiro Marques. On the discretization of degenerate sweeping processes. *Portugal. Math.*, 55(2):219–232, 1998.