

Dissipative measure differential equations in Wasserstein spaces

We present the analysis of well-posedness for so-called *Measure Differential Equations* (MDE). These are evolution equations with state belonging to the Wasserstein space $\mathcal{P}_2(\mathbf{X})$ of Borel probability measures on a Hilbert space \mathbf{X} . The vector field itself is a *probability vector field* $V : \mathcal{P}_2(\mathbf{X}) \rightarrow \mathcal{P}_2(\mathbf{TX})$. We develop a suitable notion of dissipativity for V in the (metric) Wasserstein space. Taking inspiration from the theory of dissipative operators in Hilbert spaces and of Wasserstein gradient flows of geodesically convex functionals, we define solutions to (MDE) through a suitable *Evolution Variational Inequality* (EVI). Our approach is based on a measure-theoretic version of the Explicit Euler scheme.

If time allows, we introduce a Lagrangian counterpart: a characterization of the measure-valued EVI solution as the time-dependent law of the unique solution of a corresponding evolution equation in the lifted Hilbert space $L^2(\Omega)$.

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